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The Design of MIMO Precoders Based on Decomposition Algorithms

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Abstract:

The Wireless Communication over Multiple Input and Multiple Output (MIMO) channel increases transmission rate by splitting the input data stream into a plethora of parallel data streams that are transmitted in simultaneously. The goal of precoding at the transmitter is to divide the channel into numerous unconnected subchannels so that many data streams may be sent out at the same time. This article examines a MIMO precoder's performance utilising different channel decomposition method, as well as its computational complexity in terms of the number of floating-point operations (FLOPs). Singular Value Decomposition (SVD), Geometric Mean Decomposition (GMD), LDLH, LU, Schur, QR, and Jordan decomposition are among the techniques discussed. According to simulation findings, precoding for MIMO based on QR decomposition beats all other precoding techniques based on channel decomposition in terms of BER performance and requires less FLOPs.

Keywords— MIMO, Precoding, Channel decomposition, Subchannel.

1. INTRODUCTION

To increase channel capacity and connection dependability, MIMO technology employs multiple antennas at both the transmitter and receiver. The capacity to utilize multi-path propagation for the advantage of the user is a fundamental characteristic of MIMO systems. It has the potential to boost data rates without requiring more bandwidth. As a result, MIMO improves spectral efficiency. Alamouti's first work on fundamental spatial variety [1] minimizes the damage caused by multipath propagation. Arogyaswami Paulraj and Thomas Kailath, two additional researchers, suggested the use of spatial multiplexing with MIMO in 1993. By decomposing the complicated MIMO channel into parallel unicast channels, the MIMO communication system sends data via parallel subchannels. The de-correlation of the data streams that are transmitted in simultaneously is the most important need in MIMO communications. Precoding or pre-equalization of the signal is done at the transmitter for this reason. Precoding is a beam shaping simplification that enables multi-layer transmissions in multi-antenna wireless communications.

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Increased signal strength at the receiver owing to constructive signal addition and reduced multipath fading are two advantages of beamforming [2]. In single layer beam forming, each transmitter antenna broadcasts the same signal with the appropriate weighting, maximizing the signal strength to the receiver. Multi-layer beam shaping is needed to optimise signal levels at all reception antennas at the same time.

The channel state information (CSI) is required for pre-processing or precoding at the transmitter. In order to achieve CSI at the transmitter, the channel must be fixed (non-mobile) or nearly constant for a long period of time. If the transmitter has CSI, the symbols for transmission, whether from a single user or many users, may be separated at the transmitter using pre-equalization. The MIMO channel is decomposed into parallel unicast channels using channel decomposition techniques. The SVD method is often used to breakdown a MIMO channel into parallel eigen modes, yielding a broad variety of singular values. The homogeneity of subchannel gain is influenced by the broad dynamic range of singular values (diagonal elements). Many SVD-based decomposition methods have been proposed in the literature to enhance sub channel gain uniformity [3], [4], [5]. GMD and UCD are two of the most widely used methods of uniform channel decomposition. SVD-based systems maintain the regularity of the subchannel gain. For (moderately) high SNR, the GMD scheme is asymptotically optimal in terms of channel throughput and bit error rate. Lower SNR, on the other hand, causes a reduction in the system's capability. Even at lower SNR levels, the UCD system provides more capacity. These schemes outperform SVD with power allocation in terms of error performance, but at the expense of a greater level of complexity. LDLH decomposition may be used to decrease computational complexity. The diagonal and lower triangular components of the channel matrix are utilised in this approach, which avoids computing the square roots of the diagonal elements during power allocation [6]. Despite the fact that its complexity is decreased, its BER performance is worse to that of SVD-based systems. If correct CSI is given, these decomposition methods ensure optimum power allocation. Perfect CSI, on the other hand, causes MIMO systems' feedback burden to rise. It is necessary to develop an optimum decomposition strategy that improves BER performance without increasing the number of processes.

2. System Model

As illustrated in Figure 1, a single-user MIMO system has Nt transmission antennas and Nr receive antennas. Consider F to be the precoding matrix and B to be the decoding matrix. x represents input bits for transmission. The transmitted signal after precoding is $\tilde{x} = Fs$. We assume, a flat fading MIMO channel, whose system input output relationship can be mathematically modeled as,

$$y = H * \tilde{x} + n \qquad (1)$$

At the receiver, the received signal after decoding is given by

$$\tilde{y} = B * H * \tilde{\text{x}} + B * n \qquad (2)$$

On Substituting $\tilde{x} = Fs$ in (2), we get

$$\tilde{y} = B * H * \text{F} * \text{s} + B * n \qquad (3)$$

3. Channel Decomposition for Precoding

Using channel decomposition techniques, a MIMO channel may be divided into parallel independent channels. Independent data streams may be broadcast in each of the sub channels via spatial multiplexing, allowing data rates to be enhanced without the need of extra capacity. In diversity mode, each subchannel receives the same copy of the data. In diversity mode, data transmission is more reliable. The data rate, on the other hand, is not raised.

3.1 Singular Value Decomposition

Consider a 2×2 MIMO system. The channel matrix is written as

$$H = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \qquad (4)$$

By using SVD, the channel matrix H is decomposed in to the product of two unitary matrices U and V and a diagonal matrix Σ.

$$H = U * \Sigma * V^H = \begin{bmatrix} u_1 & u_2 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} V_1^H \\ V_2^H \end{bmatrix} \qquad (5)$$

Where

$$v_1 = \begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix}, \text{v}_2 = \begin{bmatrix} v_{21} \\ v_{22} \end{bmatrix}, u_1 = \begin{bmatrix} u_{11} \\ u_{12} \end{bmatrix}, u_2 = \begin{bmatrix} u_{21} \\ u_{22} \end{bmatrix}$$

U*UH = V*VH = I and is a diagonal matrix having nonnegative singular values as its diagonal members, where U and VH are unitary matrices. rH the rank of H is defined as the number of nonzero singular values of H. When computing the rank, the goal is to find the lowest possible value between the number of independent rows and columns in H. because of which trH min (Nt,Nr). Since it defines the maximum number of independent streams that may be multiplexed at the same time, the channel matrix H rank is an essential parameter. The Precoding matrix (F) is V, decoding matrix is $U^H$and power allocation matrix is $\sum$

On substituting $F= V$,$B=U^H$ and $H=U*\sum*V^H$ in (3), we get

$$\tilde{y} = U^H * U * \Sigma * V^H * V \; * s + U^H * n \qquad (6)$$

On solving we get, $\tilde{y} = \Sigma * s + V^H * n$ (7)

3.2 Geometric Mean Decomposition

The decomposition of a channel matrix using SVD usually yields sub channels with a high condition number. The modulation/demodulation and coding/decoding operations become more complicated as a result. Consider that all of the sub channels have the same modulation. The performance of the inferior sub channel suffers significantly. As a result, weaker channels should be given greater power. As a result, there should be a compromise between BER and channel performance throughout.

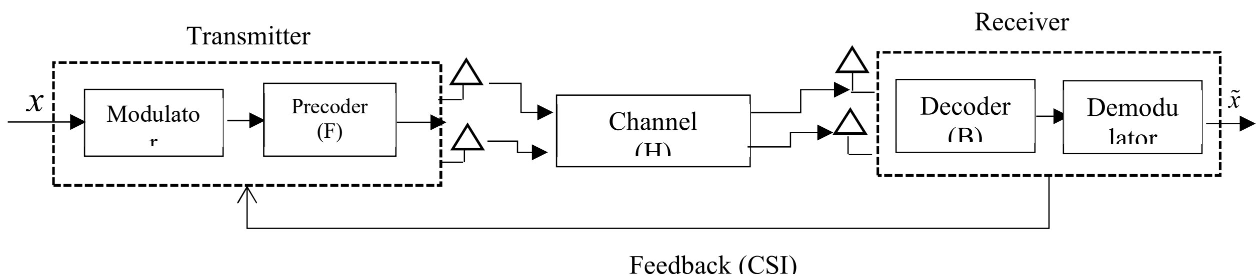

Figure. 1 System Model

The GMD is produced by doing extra calculations to equalise the SVD's diagonal elements. At high SNR levels, the GMD method performs asymptotically equivalent to the maximum likelihood detector (MLD) [3]. The channel matrix H is deconstructed as follows in GMD-based precoding:

$$H = Q\,R\,P^H \tag{8}$$

Whenever Q and P are unitary, the eigenvalues of channel matrix H have the geometric mean of R, an upper triangular matrix with equal diagonal components. The noise vector is not magnified by Q since it is unitary. The signal strength is equally divided across parallel sub channels, as shown by R's equal diagonal elements. This helps in lowering the bit error rate (BER). As a result, P is the precoding matrix and QH is the decoding matrix.

On substituting F= P, B=Q^H and H=QRP^Hin (3)

$$y = Q^H * Q * R * P^H * P * s + Q^H * n \tag{9}$$

On solving we get, $y = R * s + Q^H * n$ (10)

3.3 LDL^H Decomposition

Using LDL^H decomposition, a Hermitian matrix can be factorized as follows

$$H = LDL^H \tag{11}$$

Where L is a Lower triangular matrix and D is a diagonal matrix. Generally channel matrix H is not

Hermitian.To make it Hermitian, a matched filter H^H can be used at the receiver [6].

$$HH^H = LDL^H \tag{12}$$

Where $L \in \mathbb{C}^{N_t \times N_t}, D = diag\{d_1, d_2 .... d_k, 0_{N_t\text{-}K}\}$ for each $d_i \geq 0$ and $0_{N_t\text{-}K}$ is a row vector with N_t-K zeros. From (19) we can derive,

$$D = (L^{-1}H^H)H(L^{-1})^H \tag{13}$$

The precoding matrix *is* $F = (L^{-1})^H$and decoding matrix is $B = D^{-(1/2)}L^{-1}H^H$

On substituting$F = (L^{-1})^H$ and $B = D^{-(1/2)}L^{-1}H^H$ in (3)

$$y = D^{-(1/2)}L^{-1}H^H * H * \left(L^{-1}\right)^H * P * s + D^{-(1/2)}L^{-1}H^H * n \tag{14}$$

On solving we get, $y = D^{(1/2)}P * s + n_L$ (15)

Where $n_L = D^{-(1/2)}L^{-1}H^H * n$

3.4 QR Decomposition

The QR decomposition decomposes the matrix into an orthogonal and triangular matrix product. A channel matrix H's QR decomposition is given as

$$H = Q * R \tag{16}$$

Where H is of dimension m-by-n. It produces an m-by-n upper triangular matrix R and an m-by-m unitary matrix Q so that $Q*Q^H = I$. The precoding matrix is R^H and decoding matrix is Q^H

On substitutingF= R^H, B=Q^H and H=Q*Rin (3)

$$y = Q^H * Q * R * R^H * s + Q^H * n \tag{17}$$

On solving we get,

$$y = s + Q^H * n \tag{18}$$

3.5 LU Decomposition

Assume that A is a four-by-four matrices. LU factorization divides A into two components, a lower triangular matrix L and an upper triangular matrix U, where the row and/or column orderings and permutations are correct.,

$$A = L * U \tag{19}$$

The elements above the diagonal in the lower triangular matrix are all zero, whereas the elements below the leading diagonal are all zero in the upper triangular matrix. For instance, the LU decomposition of a 3-by-3 matrix A looks like this:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} \tag{20}$$

The channel matrix H can be decomposed as

$$H=L*U \quad (21)$$

In this scheme, the precoding matrix is $U^H$and decoding matrix is L^H

On substituting, F= U^H and B=L^H and H=L*Uin (3)

$$\tilde{y} = L^H * L * U * U^H * s + L^H * n \quad (22)$$

On solving we get, $\tilde{y} = s + L^H * n$ (23)

3.6 Schur Decomposition

There is an upper triangular matrix T=U*AU for every n-by-n matrix A (i.e. U*=U-1 or U*U=I) (i.e., everything below the main diagonal is zero). Additionally, the Eigenvalues of A may be found on T's major diagonal, as shown below. The channel matrices H may be broken down into the following components:

$$H=U * T * U^H \quad (24)$$

Where T is a quasitriangular schur matrix. U and U' are unitary matrices. The precoding matrix is T^H.

On substituting$F= T^H$ and $H=U * T * U^H$ in (3)

$$y = U * T * U^H * T^H * s + n \quad (25)$$

On solving we get, $y = s + n$ (26)

3.7 Jordan Decomposition

Jordan decomposition computes the Jordan Canonical/Normal Form (JCF) of the matrix A. The matrix must be known exactly. Hence, its elements must be integers or ratios of small integers. Any error in the input matrix can completely change its JCF. It also computes the similarity transformation, *V*, so that *V\A*V = J*. The columns of V are the generalized eigenvectors.The channel matrix H can be decomposed as

$$H=V*J*V^{-1} \quad (27)$$

In this scheme precoder matrix and decoder matrix are *V* and

V^{-1}respectively. On substituting $F= V$, $B=V^{-1}$ and $H=V*J*V^{-1}$in (3)

$$y = V^{-1} * V * J * V^{-1} * V * s + V^{-1} * n \quad (28)$$

On solving we get, $y = J * s + V^{-1} * n$ (29)

4. Performance Analysis

4.1 SER analysis

Simulations were performed for the design of precoder matrix based on each decomposition schemes discussed so far. Their corresponding SER performance has been analyzed. A 2×2 MIMO system with N_t=2 transmitter antennas and $N_r=2$ receiver antennas, and a flat fading channel is considered. The baseband modulation scheme used is Binary phase shift keying. The SER performances of all the decomposition methods were compared in Figure 2. The results were plotted by averaging over 20 independent channel realizations. The simulation result indicates that precoding methods based on QR channel decomposition performs better when compared to other channel decomposition methods.

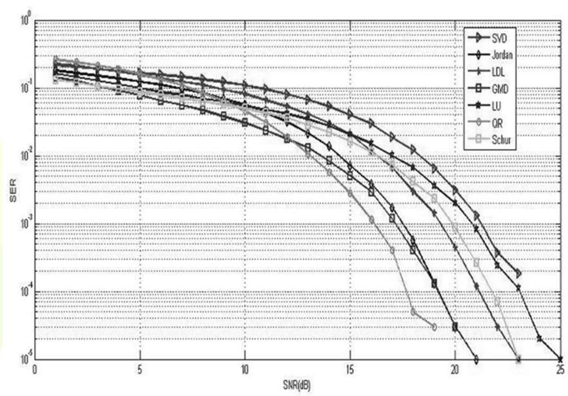

Figure.2 Comparison of SER plots

4.2 Complexity Analysis

For a mn channel matrix, the computational cost of all seven decomposition methods has been investigated. The amount of flops needed for each decomposition method is shown in Table 1. The fact that QR decomposition only needs the calculation of a single unitary matrix for decoding reflects its simplicity. In contrast, SVD requires the calculation of two unitary matrices. The computational cost of GMD, which is a mix of SVD and QR decomposition, is considerable.

It is evident that the number of operations required

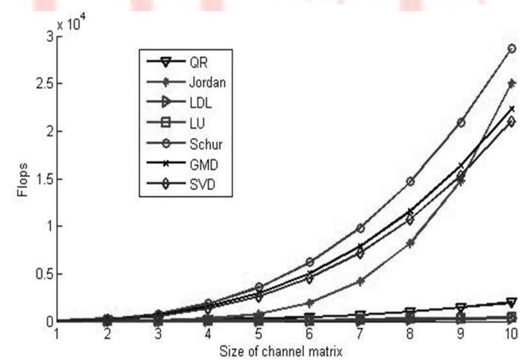

Figure. 3 Size of channel matrix versus no. of Flops required

for Schur decomposition is 86 times of what is requiredfor LDL decomposition. Similarly LU decomposition requires sum additional operations when compared to LDL. Though not immediately perceivable jordan decomposition is computationally heavy when compared to LU decomposition.

Table 1: Computations Involved In Decomposition Schemes.

Decomposition scheme	No. of flops required for (mxn) matrix
SVD	$9m^3 + 8mn^2 + 4m^2n$
GMD	Flops of SVD+ [$(4m+n)m + 4nm + 4n^2 + n$]
LDL^H	$n^3 / 3$
LU	$n^3 / 3 + n^2 - n / 3$
Schur	$\frac{86}{3} n^3$
QR	$2mn^2$
Jordan	$n^3 / 2 + n^2 / 2$

5. Conclusion

In this paper, the SER performance of MIMO precoder using various channel decomposition schemes have been evaluated and also compared. Simulation results demonstrate that precoder design based on the QR decomposition outperforms all the other decomposition methods considered with respect to SER performance. In addition, The computational complexity in terms of the number of FLOPs of the precoder based on QR decomposition is lower than that required for the other decomposition schemes considered. For a 4×4 MIMO sysytem the number of FLOPs involved in QR decomposition is 128, while conventional SVD requires 1344 FLOPs. From the simulation results, it can be concluded that precoders based on QR decomposition scheme have reduced complexity with better SER performance.

References

[1] Siavash M. Alamouti, ‘A Simple Transmit Diversity Technique for Wireless Communications’ IEEE Journal On Select Areas In Communications,, Vol. 16, No. 8, pp.1451-1458, Oct. 1998.

[2] Monica Lamba and Charanjit Singh ‘Challenges and Future Direction for MIMO Communication System’ Sop Transactions On Wireless Communications ISSN(Print): 2373-8618 ISSN(Online): 2373-8626 Vol 1, No.2, pp.42-50, July 2014.

[3] Wei-Da Chen and Yin-Tsung Hwang ‘A constant throughput geometric mean decomposition scheme design for wireless MIMO precoding’ IEEE Transactions On Vehicular Technology, Vol. 62, NO. 5, pp.2080-2090, June 2013.

[4] Yin-Tsung Hwang, Wei-Da Chen and Cheng-Ru Hong ‘A low complexity geometric mean decomposition computing scheme and its high throughput VLSI implementation’ IEEE Transactions On Circuits And Systems, Vol. 61, No.4, pp.1170-1182,April 2014.

[5] Yi Jiang, Jian Li and William W. Hager, ‘Uniform channel decomposition for MIMO communications’ IEEE Transactions On Signal Processing, Vol. 53, No. 11, pp.4283-4294, Nov.2005.

[6] Che-Chen Chou and Jen-Ming Wu, ‘Low-complexity MIMO precoder design with LDLHchannel decomposition’ IEEE Transactions On Vehicular Technology, Vol. 60, No. 5, pp.2368-2372, June 2011.

[7] Tim Brown, Elisabeth De Carvalho, Persefoni Kyritsi ‘Practical Guide to the Mimo Radio Channel With Matlab Examples’, John Wiley & Sons, Ltd., Publication, Feb. 2012.

[8] Taewon Hwang and Younggap Kwon, ‘Root mean square decomposition for EST-based spatial multiplexing systems’ IEEE Transactions On Signal Processing, Vol. 60, No. 3, pp.1295-1306, March 2012.

[9] Le-Nam Tran, Markku Juntti and Een-Kee Hong, ‘On the Precoder Design for Block Diagonalized MIMO Broadcast Channels’ IEEE Communications Letters, Vol. 16, No. 8,pp.1165-1168, Aug. 2012.

[10] Keke Zu, Rodrigo C. de Lamare, and Martin Haardt, ‘Generalized design of low-complexity block diagonalization type precoding algorithms for Multiuser MIMO systems’ IEEE Transactions On Communications, Vol. 61, No. 10, pp.4232-4242, Oct. 2013.

[11] Jun Tong, Peter J. Schreier, and Steven R. Weller, ‘Linear Precoding for MIMO Systems with Low-Complexity Receivers’ IEEE Transactions On Wireless Communications, Vol. 11, No. 8,pp.2828-2837, Aug. 2012.